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Private Information and Optimal Infant Industry Protection *

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Abstract

We study infant industry protection using a dynamic model in which the industry's cost is initially higher than that of foreign competitors. The industry can stochastically lower its cost via learning by doing. Whether the industry has transitioned to low cost is private information. We use a mechanism-design approach to induce the industry to reveal its true cost. We show that (i) the optimal protection, measured by infant industry output, declines over time and is less than that under public information, (ii) the optimal protection policy is time consistent under public information but not under private information, (iii) the optimal protection policy can be implemented with minimal information requirements, and (iv) a government with a limited budget can use a simple approach to choose which industries to protect.

JEL codes: F10, F13, O25, D82.

Keywords: Protection, Infant Industry, Private Information, Mechanism Design, Time Consistency.

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1 Introduction

Protection of infant industries is perhaps the longest-lived exception to free trade. Examples of protection range from those in the 19th century in the U.S. for steel rail (Head, 1994) and tinplate (Irwin, 2000) to more recent ones, such as the chemical industry in South Korea (Choi and Levchenko, 2021). The rationale for infant industry protection is that a newer, smaller domestic industry cannot survive against mature foreign competitors who have a superior technology. Protection provides the infant industry the time to develop so it can compete in the world market.[1]

In developing economies, there are of course many infant industries, so a question then is what are the prerequisites for protecting an industry. The answer is that the protection policy must pass the Mill and Bastable tests: "The Mill test requires that the protected sector can eventually survive international competition without protection, whereas the Bastable test requires that the discounted future benefits compensate the present costs of protection," see Harrison and Rodriguez-Clare (2010).[2] In reality, assessing the costs and benefits is fraught with private information problems, especially in a developing economy. The infant industry's cost of production at any point in time is known to the industry, but not the government.[3]

Our focus is on the private information problem in infant industry protection. The infant industry knows when it has reduced its cost and is ready to compete with foreign firms. When the cost is private information, the industry has an incentive to say it has not reduced its cost so that it can continue to receive protection. We use a dynamic mechanism-design approach to design an efficient protection policy that results in the infant industry truthfully reporting its cost.

We develop a model where foreign firms have zero marginal cost of production

[1]Government support for domestic producers to compete against foreign competitors was advocated by Yarranton (1677). The infant industry argument dates back to Hamilton (1791) and then to Rae (1834), List (1841), and Mill (1848). For an intellectual history of the argument see Irwin (1996).

[2]The answer has been refined over many years, from Mill (1848) and Bastable (1887, 1921) to Meade (1955), Kemp (1960), and Succar (1987).

[3]For instance, items for personal entertainment, such as big screen TVs, can be recorded as production equipment thereby giving the illusion of high cost, so government would not know the true cost of the industry. Even in developed countries, Breyer (1982, p. 109-110) observed that in setting tire standards, the National Highway Traffic Safety Administration needs to know the cost of developing practical tests for tire qualities, such as blow-out resistance and stopping distance. When firms provide the estimates, "it was easy for a firm...to produce a high cost estimate..."

and where a domestic industry initially has a positive marginal cost c for producing the same good. There is a downward sloping domestic demand curve for the good. Without protection the foreign firms will serve the entire domestic market. We assume that there is an externality associated with free trade; we capture this via a social cost that is increasing in imports. The domestic industry's cost may stochastically transition from c to zero at a Poisson arrival rate that is increasing in domestic output. This stochastic process has the feature that the expected marginal cost of the industry declines over time as long as it continues to produce; i.e., the process embodies stochastic learning by doing. After the transition the industry's cost is zero forever. The government knows the initial cost c, but after the initial period the government does not know the industry's true cost. The time at which the industry transitions from c to zero is private information and random.

We restrict the demand, learning by doing, and social cost such that the Bastable test is satisfied. The question in our paper is how to protect the infant industry optimally when the industry has private information. Our mechanism has access to all instruments—tariffs, domestic production/import quotas, rewards, taxes, and subsidies—in order to maximize social welfare. In our dynamic setting, truth telling is achieved by conditioning future payoffs on the history of reports. That is, our optimal mechanism offers a higher reward for reporting an early transition to zero cost than for reporting a late transition.

Our results are as follows: The optimal protection policy is a precommitted sequence, from time 0 to the infinite future, of (i) domestic output and import quotas before the report of transition to zero cost, (ii) a per-unit subsidy c, financed by a tax on consumers, to cover the gap between domestic and foreign costs of production before the report of transition to zero cost, and (iii) a reward at the time when the industry reports a transition to zero cost. After the transition to zero cost, the policy is straightforward: The domestic industry receives no subsidies and serves the entire domestic market. Under private information, the optimal domestic output declines (or, the optimal import quota increases) over time, before the transition to zero cost. Facing declining subsidies, the domestic industry is incentivized via a reward to never postpone its report of transition to zero cost. In contrast, under public information where the government can observe the transition to zero cost, the optimal protection policy before the transition is a constant level of domestic output with a per-unit subsidy c; there is no reward at the time of transition. Furthermore, the protection under public information is more generous: The optimal import quota is lower than that under private information at every point in time. This is because protection is more costly under private information since the government has to reward the domestic industry to induce truth telling.

We show that the optimal allocation under private information can be achieved with a simple implementation. The government provides a fund upfront to the infant industry and requires the industry to choose a production level from an interval that is bounded below by the optimal domestic output. Based on the industry's choice of production (which is observable), the government determines the consumption tax and import quota, which pin down the price. The information requirement on our implementation is minimal: The industry receives no subsidy or reward other than the initial fund and is not required to report its transition to zero cost.

Is it possible that the policy fails in the sense that it never produces a viable domestic industry that can compete internationally? In other words, does the policy pass the Mill test? Our model is stochastic, so passing the Mill test is a probabilistic event. Under public information, with constant output each period and stochastic learning by doing, the domestic industry will eventually be able to compete internationally with probability 1. Under private information, this is not the case. The optimal policy is a declining path of domestic output, and the probability of eventual transition to zero cost, although positive, is below 1. So, the protection policy cannot guarantee that the domestic industry will be able to compete internationally.

The optimal protection policy under private information is not time consistent. In our model, time inconsistency arises from incentive compatibility constraints. Promised subsidies after t have negative effects on incentives before t, which have to be taken into account by the time-0 government, but not by the time-t government. As a result, the time-0 government is less willing to protect after t than the time-t government. A future government would embark on a new path as if it were starting at time 0. This gap between time-0 and future governments does not arise under public information since the transition is observable and incentive constraints are not an issue. Hence, the optimal policy under public information is time consistent.[4]

By restricting the mechanism to choose only from stationary policies, as in the public-information game of Matsuyama (1990), we deliver a time-consistent policy under private information. The time-consistent domestic output is constant over time as long as the domestic industry's reported cost is high. With constant output in each period, stochastic learning by doing implies that the domestic industry will eventually transition to zero cost and be able to compete internationally with probability 1. The time-consistent policy is suboptimal since it is in the feasible set of policies for the unrestricted mechanism. That is, even though the time-consistent policy passes the Mill test with probability 1, it results in lower welfare. Furthermore, the time-consistent policy offers less protection than the public-information policy since the industry has to be incentivized under private information to report the transition truthfully, which increases the cost of protection.

How would a government with a limited budget choose between industries asking for protection? An industry in our model is defined by four items: its initial cost relative to the foreign competitor, the parameters of its stochastic learning-by-doing function, the social cost of imports, and the demand for its product. Given the budget and the set of industries asking for protection, we show that the shadow value of government's resources helps determine which industries should be protected. Furthermore, our one-industry mechanism delivers the optimal protection policy for each protected industry when each industry's cost is magnified by the shadow value of the government's resources. We show that an industry with a higher initial cost of production, all else equal, is protected less, i.e., lower domestic output and higher import quota. This is because the production subsidy increases with c. In contrast, Costinot, Donaldson, Vogel, and Werning (2015) and Bartelme, Costinot, Donaldson, and Rodriguez-Clare (2021) show that tariffs are uniform among importing industries despite different levels of comparative disadvantage. In their model, imports have no social costs and specialization is complete, so all importing industries have zero output and receive zero subsidies.

[4] In both cases the protection is contingent on the industry's "effort" (domestic output) in cost reduction, which is observable by the government. Tornell (1991) notes that contingent policy resolves the time inconsistency issue under public information. With private information, a policy contingent on observables does not resolve the time inconsistency.

Finally, our paper also contributes to the methodology for solving persistent-private-information models. The standard method in this literature (e.g., Fernandes and Phelan, 2000) is to formulate the principal's problem recursively and use a vector of agent's continuation utilities as the state variable. The first-order approach (e.g., Williams, 2011; Farhi and Werning, 2013) reduces the state vector to a pair: continuation utilities of only the truth teller and his nearest neighbor in the type space. In contrast, our state variable is *not* continuation utilities, but the domestic industry's cumulative probability of transition to zero cost. We can identify all of the binding incentive constraints and substitute them into the principal's objective function.

A few remarks are in order here. First, our model is about how, not why, to protect the infant industry. The structural parameters are restricted to satisfy the Bastable condition so that the industry is worth protecting. The mechanism-design approach delivers how to protect the infant industry optimally. Second, our mechanism does not restrict the set of available instruments. In contrast, Bardhan (1971) and Melitz (2005) study infant industry protection under public information by comparing the effectiveness of specific policy instruments.

2 Model

There is a unit measure of buyers with the inverse demand function $p(Q)$, where Q is the total quantity of the good. This could be a derived demand for an intermediate good or final demand by consumers. A domestic industry can produce the good at cost cq, where $q \geq 0$ is the quantity produced by the domestic industry and $c > 0$ is the cost per unit. Foreign firms can produce the same good at zero cost. In a laissez-faire equilibrium the foreign firms would drive out the domestic firms.

We assume that imports q^f imply a social cost $\Gamma(q^f) \geq 0$, so there is a reason to protect the domestic industry. We impose the following assumption on Γ:

ASSUMPTION 1 *$\Gamma(q^f)$ is increasing and strictly convex in q^f, $\Gamma(0) = 0$, and $\Gamma'(0) = 0$.*

Learning by doing The domestic industry's cost may stochastically transition from c to zero (absorbing state) at arrival rate $\pi(q)$ that satisfies:

ASSUMPTION 2 *$\pi(q)$ is increasing and strictly concave in q, $\pi(0) = 0$, and $\pi'(0) < \infty$.*

Let Ω_t denote the event that the domestic industry's cost remains high at t; i.e., no transition has occurred until t. *Conditional* on Ω_t, the probability of a transition during time interval $(t, t+dt]$ is $\pi(q_t)dt$, where $dt > 0$ is small. Denote the *unconditional* (i.e., as of time 0) probability of Ω_t as $\Pr(\Omega_t)$; $\Pr(\Omega_0) = 1$. The unconditional probability decreases at the rate $\pi(q_t)$, so $\Pr(\Omega_t) = e^{-\Pi_t}$, where $\Pi_t \equiv \int_0^t \pi(q_s)ds$ is the cumulative transition up to t. The domestic industry's marginal cost at time t, c_t, equals 0 with unconditional probability $1 - e^{-\Pi_t}$. Its mean $E[c_t] = ce^{-\Pi_t}$ is monotonically decreasing over time. Thus, our specification is one of stochastic learning by doing. The growth rate of $E[c_t]$ is $\frac{dE[c_t]/dt}{E[c_t]} = -\pi(q_t)$, so the cost cannot be reduced further if the domestic industry stops production.

Our environment before the transition is time invariant. That is, conditional on event Ω_t, the environment at t is identical to that at time 0. In particular, the transition rate at t, $\pi(q_t)$, depends only on q_t; the cumulative production before t does not affect the *conditional* probability of transition in the interval $(t, t + dt]$, which resembles the memoryless property of stationary Markov processes. Note that the time-invariant nature of the physical environment does not necessarily imply that the optimal policy is time invariant.

Two remarks are in order here. First, the externality in our model is the social cost of imports, instead of social benefit of domestic production. Although the latter is common in the literature, our modeling choice is in the spirit of Mill (1848):

> *...it is essential that the protection should be confined to cases in which there is ground of assurance that the industry which it fosters will after a time be able to dispense with it; nor should the domestic producers ever be allowed to expect that it will be continued to them beyond the time for a fair trial of what they are capable of accomplishing.*[5]

Had we modeled the externality as social benefits of domestic production, the optimal policy would prescribe a subsidy to the domestic industry forever. Put differently, in a dynamic model, the subsidies would continue even after the domestic industry is ready to compete internationally and there would be no such thing as temporary protection in Mill's sense, by construction.

[5] The statement in its entirety is reprinted in Irwin (1996). Juhasz (2018) examines a case of "natural" protection for the cotton industry in France during the Napoleonic Wars and documents a reduction in French firms' marginal cost.

Second, there is no contradiction between the memoryless property and learning by doing. The independence between the past and future, *conditional* on the current state, is a property of models with a state variable and a Markovian structure. For example, consider a stochastic version of the learning-by-doing model in Arrow (1962): A firm's productivity depends on the stock of aggregate capital, which is the sum of past investments and random shocks. The capital stock is a state variable in this model because future dynamics depend on the history through the current stock of capital. Since a history with large investments but bad shocks can result in the same capital stock as another history with small investments but good shocks, the two histories imply the same productivity in the future despite different cumulative investments. Similarly, in our model, the state at time t is the infant industry's cost. Conditional on cost c at t, the probability of transition to zero cost after t is independent of cumulative output until t. In contrast, the *unconditional* probability of the transition, $1 - e^{-\Pi_t}$, depends on the history and increases over time due to learning by doing.

2.1 Protection under public information

As a benchmark, consider the environment with public information. The government knows when the domestic industry transitions to zero cost and precommits to a protection policy at time 0 through a direct mechanism by choosing paths of several variables. The government

1. provides a subsidy τ_t to the domestic industry and asks it to produce q_t;
2. provides a subsidy τ_t^f to the foreign firm and asks it to produce q_t^f; and
3. sets a price p_t per unit to be paid by each consumer and collects a tax $\tau_t + \tau_t^f$.

While the subsidy τ_t in each period helps the domestic industry to compete with foreign firms, τ_t^f can be negative, in which case a natural interpretation is tariff. Note that the mechanism is not restricted to choosing one instrument at a time. It can simultaneously use tariffs, import quotas, taxes, and production subsidies. The constraints on the direct mechanism are, for all t

1. nonnegative profits for the domestic industry—$\tau_t + p_t q_t \geq c_t q_t$—and

2. nonnegative profits for foreign firms—$\tau_t^f + p_t q_t^f \geq 0$.

The government's utility flow is the consumer surplus minus social cost:

$$\int_0^{q_t+q_t^f} p(Q)dQ - p_t(q_t + q_t^f) - (\tau_t + \tau_t^f) - \Gamma(q_t^f).$$

(In Appendix B, we extend the government's objective function to also include the domestic industry's payoff and show that our results are still valid.) Given that the government has access to taxes and subsidies in our mechanism, the price p_t is redundant. We can define an alternative tax/subsidy $\tilde{\tau}_t = \tau_t + p_t q_t$ and $\tilde{\tau}_t^f = \tau_t^f + p_t q_t^f$ and set the price to zero; the alternative would yield the same sequence of import quota and domestic output.

Remark 1 *Without loss of generality we can set $p_t = 0$ with τ_t and τ_t^f adjusted accordingly for all $t \geq 0$.*

Remark 2 *With $p_t = 0$, nonnegative profits for foreign firms imply $\tau_t^f \geq 0, \forall t$. Recall that positive τ^f implies a subsidy to the foreign firms while negative τ^f implies a tariff. It is easy to see that it is suboptimal to subsidize the foreign firms with a tax on domestic consumers; hence, $\tau_t^f = 0$. In general, $\tau_t^f = -p_t q_t^f < 0$ whenever $p_t > 0$ so that the foreign firms' rents are completely extracted.*

Remark 3 *With $p_t = 0$ and $\tau_t^f = 0$, nonnegative profits for the domestic industry imply $\tau_t \geq c q_t$ before the transition to zero cost. It is suboptimal to provide rents to the domestic industry—$\tau_t - c q_t > 0$—financed by a tax τ_t on consumers. The direct mechanism can internalize the learning benefits and social costs through the sequence of domestic output q_t and imports q_t^f. Thus, $\tau_t = c q_t$ before the transition.*

The above three remarks imply that even though the mechanism has access to all instruments, a subset of instruments—domestic production/import quota and subsidy to the infant industry financed by a tax on consumers—is sufficient to describe the optimal protection policy. Other instruments are redundant. For the rest of this section, we set $p_t = 0$ and $\tau_t^f = 0 \ \forall t$, and $\tau_t = c q_t$ before the transition.

It is useful to divide the government's problem into two parts: (i) after the domestic industry has transitioned to zero cost and (ii) before the domestic industry has transitioned.

After the transition, the domestic industry and the foreign firms are equally efficient: The marginal cost is 0 for both. The government's problem is a sequence of *static* problems, each of which is to

$$\max_{q,q^f,\tau} \int_0^{q+q^f} p(Q)dQ - \tau - \Gamma(q^f) \quad s.t. \quad \tau \geq 0.$$

Since the domestic firm need not be subsidized after the transition, it is optimal to set $\tau = 0$. Furthermore, since imports impose a social cost, it is optimal to set the import quota to zero and $q = p^{-1}(0)$: The domestic output is such that the marginal utility of consumption equals the marginal cost, which is 0. Denote the optimal value in the above problem as

$$S \equiv \int_0^{p^{-1}(0)} p(Q)dQ.$$

Note that S is the flow of consumer surplus in each period after the transition and it is also the flow of social surplus.

Before the transition to zero cost, the government's infinite-horizon problem is

$$\max_{q_t,q_t^f} \quad E\left[\int_0^T e^{-rt}\left(\int_0^{q_t+q_t^f} p(Q)dQ - cq_t - \Gamma(q_t^f)\right)dt + \int_T^\infty e^{-rt}Sdt\right], \tag{1}$$

where T is the random transition time, and $\int_0^{q_t+q_t^f} p(Q)dQ - cq_t - \Gamma(q_t^f)$ and S are the government's payoff flows at $t < T$ and $t \geq T$, respectively. Note that q_t^f affects only the payoff flow at t, while q_t affects not only the payoff flow but also the random variable T. Consequently, the optimal q_t^f before the transition is a solution to a static optimization problem, while the optimal q_t requires a dynamic analysis and is the focus of this paper.

Import quota For any given q_t, the optimal import quota is a solution to

$$\max_{q_t^f} \quad \int_0^{q_t+q_t^f} p(Q)dQ - \Gamma(q_t^f).$$

The first-order condition for the optimal q_t^{f*} is

$$\Gamma'(q_t^{f*}) = p(q_t + q_t^{f*}), \tag{2}$$

which implicitly defines q_t^{f*} as a *stationary* function of q_t. We denote this function as $q_t^{f*} = q^f(q_t)$.

LEMMA 1 (Import quota) *$q^f(q)$ is decreasing in q but total quantity $q + q^f(q)$ is increasing in q.*

The total $q + q^f(q)$ varies with q. The substitution between q and q^f is imperfect in the optimal protection policy even though consumers view the domestic output and imports as perfect substitutes. When q is decreased, q^f is increased, but this increase is less than one-for-one because the *marginal* social cost Γ' increases with q^f.

Static protection When the government cares about only the current payoff flow, $\int_0^{q+q^f(q)} p(Q)dQ - cq$, the optimal domestic output q^{stat} is a solution to

$$p(q + q^f(q)) = c. \tag{3}$$

The government internalizes the social cost of imports, but not the stochastic learning-by-doing benefits.

Dynamic protection Let

$$U(q) \equiv \int_0^{q+q^f(q)} p(Q)dQ.$$

Note that $U(q)$ is just a generalization of S and describes the consumers' utility flow in each period *before* the transition to zero cost.

After changing the order of integration, we can rewrite the objective in (1) as

$$\begin{aligned}
& \int_0^\infty e^{-rt} \left[\Pr(T \geq t) \left(U(q_t) - \Gamma(q^f(q_t)) - cq_t\right) + (1 - \Pr(T \geq t))S\right] dt \\
= \; & \frac{S}{r} + \int_0^\infty e^{-rt} \Pr(T \geq t) \left(U(q_t) - S - \Gamma(q^f(q_t)) - cq_t\right) dt \\
= \; & \frac{S}{r} - \int_0^\infty e^{-rt - \Pi_t} \left(S - U(q_t) + \Gamma(q^f(q_t)) + cq_t\right) dt, \qquad (4)
\end{aligned}$$

where $\Pr(T \geq t) \equiv e^{-\Pi_t}$ is the probability that the transition has *not* arrived until t, and $S - U(q_t)$ is the deviation of consumers' utility flow prior to transition relative to the flow post transition. We note two features of (4). First, the optimal q_t involves a dynamic tradeoff due to learning by doing: q_t affects not only the current payoff, but also future payoffs through probabilities $e^{-\Pi_s}, \forall s > t$. Second, because $\Gamma(q^f(q_t))$ and cq_t in (4) are, respectively, the deviations of social cost of imports and production subsidy from their post-transition counterparts, 0 and 0, the objective before transition is written as the post-transition value, $\frac{S}{r}$, minus three losses: 1) deviation of consumers' utility, 2) social cost of imports, and 3) production subsidy.

The first two losses always show up together in our analysis. Denote their sum by

$$L(q) \equiv S - U(q) + \Gamma(q^f(q)) \geq 0.$$

It is easy to show that $L(q)$ is convex and decreasing in q: The Envelope theorem implies that $L'(q) = -p(q + q^f(q)) = -\Gamma'(q^f(q)) < 0$, where $-\Gamma'$ is increasing in q because Lemma 1 shows $q^f(q))$ is decreasing in q. Higher q increases consumers' utility by $p(q + q^f(q))$; equivalently, higher q decreases the social cost by $\Gamma'(q^f(q))$. Furthermore, $L(p^{-1}(0)) = 0$; i.e., when the domestic output is $p^{-1}(0)$, which occurs after the transition to zero cost, there is no loss and the flow of social surplus is S.

Before characterizing the dynamic protection policy, it is useful to know whether the domestic industry is worth protecting, i.e., whether the benefit from protection exceeds the cost. This amounts to checking whether the industry satisfies the

$$\text{Bastable condition: } c < \bar{c} \equiv -L'(0) + \pi'(0)\frac{L(0)}{r}. \qquad (5)$$

To understand the condition, consider a temporary variation around the scenario where the domestic industry produces zero output forever: Increase q_0 from 0 to $\epsilon > 0$ but fix $q_t = 0$ for all $t > 0$. A marginal increase in q_0 costs the government c to protect. The benefits are: (i) an immediate marginal social gain of $-L'(0)$ since imports at $t = 0$ are decreased and (ii) an increase in the probability of transition resulting in a gain *forever* yielding the second term $\pi'(0)\frac{L(0)}{r}$.[6]

Recall from Assumption 2 that $\pi'(0) < \infty$. If $\pi'(0) = \infty$, then $\bar{c} = \infty$ and the Bastable condition will be satisfied by the industry no matter how high its cost is. For the rest of this section, we will assume that (5) is satisfied: $c < \bar{c} < \infty$.

Theorem 1 below characterizes the optimal policy under public information. Since $\frac{S}{r}$ is a constant, we will minimize the sum of the three losses in (4).

Theorem 1 (Permanent protection under public information) *Assume $c < \bar{c}$.*

1. *Before the transition to zero cost, the optimal domestic output is time invariant: $q_t = q^{pub} > 0$, where*

$$q^{pub} \in \arg\min \frac{L(q) + cq}{r + \pi(q)} = \frac{S - U(q) + \Gamma(q^f(q)) + cq}{r + \pi(q)}. \tag{6}$$

2. $q^{pub} > q^{stat}$.

Theorem 1 has four implications. First, the optimal protection policy is time invariant because if the domestic industry has not transitioned to zero cost until t, the continuation problem faced by the government at t is identical to that at time 0. Hence, whatever is optimal at time 0 will continue to be optimal at time t, conditional on high cost at t. Thus, the protection policy under public information is time consistent. However, as we show in the next section, if the domestic industry *privately* observes the transition to low cost, this is no longer the case.

Second, since the domestic industry's cost is public information, optimality combined with the stationary nature of the environment requires that the protection continues so long as the industry has not transitioned to zero cost. The perpetuity of protection (conditional on high cost) implies that the transition will occur eventually

[6] The definition of $\bar{c}$ includes the function $L(\cdot)$, which has import quota q^f that depends (endogenously) on domestic output. However, the import quota at zero domestic output is easy to compute from the primitives of the model using equation (2): $\Gamma'(q^f) = p(q^f)$.

with probability 1. In other words, the protection policy would pass the Mill test eventually and the domestic industry would be able to compete internationally. (As noted earlier, passing the Mill test is a probabilistic event in our model since learning by doing is stochastic.)

Third, the domestic industry is offered protection (i.e., $q^{pub} > 0$) if and only if $c < \bar{c}$. That is, if the domestic industry is "too inefficient" relative to the foreign firms, then it is optimal to let the imports serve the entire domestic demand forever. In the minimization problem (6), higher c raises the cost of protection and reduces q^{pub}. If $c \geq \bar{c}$, then the loss in (6) is weakly increasing at $q = 0$, so $0 \in \arg\min \frac{L(q)+cq}{r+\pi(q)}$.[7]

Fourth, q^{pub} is higher than q^{stat} because of learning by doing. Static and dynamic optimization under public information have the same cost of protection: Higher domestic output means higher subsidy to the industry. However, there is only one benefit of higher q in the static case: reduction in social cost by $\Gamma'(q^f(q))$. In the dynamic case, higher q also leads to faster learning by doing (higher $\pi(q)$) and decreases the present value of the social loss. This additional benefit raises the optimal protection level q^{pub} above q^{stat}.

3 Dynamic protection under private information

Our model setup is identical to that in Section 2 except for the presence of private information. We incorporate private information as follows: The government knows the domestic industry's initial cost c but does not observe when the transition to zero cost occurs. The transition is private information to the domestic industry. Note that zero-cost is an absorbing state, so the private information is persistent. The

[7] The Bastable condition can also be seen by considering a permanent variation around a stationary policy: Increase domestic output from 0 $\forall t$ to $\epsilon > 0$ $\forall t$. Since the loss is $\frac{L(q)+cq}{r+\pi(q)}$ in a stationary policy, the impact of the increase in domestic output is:

$$\begin{aligned} \frac{d\frac{L(q)+cq}{r+\pi(q)}}{dq}|_{q=0} &= \frac{c}{r+\pi(q)}|_{q=0} - \left(\frac{-L'(q)}{r+\pi(q)}|_{q=0} + \frac{(L(q)+cq)\pi'(q)}{(r+\pi(q))^2}|_{q=0} \right), \\ &= \frac{c}{r} - \left(\frac{-L'(0)}{r} + \frac{L(0)\pi'(0)}{r^2} \right). \end{aligned}$$

The three terms correspond to the cost and two benefits in the temporary variation noted above, but with an amplification factor $\frac{1}{r}$ due to the permanent increase in domestic output. The Bastable condition is not affected by this factor.

government observes the domestic output and imports.

The government precommits to a protection policy at time 0 through a direct mechanism by choosing paths of several variables. The government

1. provides a subsidy τ_t to the domestic industry and asks it to produce q_t;
2. provides a subsidy τ_t^f to the foreign firm and asks it to produce q_t^f;
3. sets a price p_t for consumers and collects a tax $\tau_t + \tau_t^f$; and
4. provides a one-time reward M_t to the domestic industry, financed by a lump-sum tax, *if* the domestic industry reports a transition to zero cost at t.[8]

Under the revelation principle, we can focus on direct mechanisms that are *incentive compatible*, i.e., protection polices that induce the domestic industry to report its transition time truthfully. The constraints on the mechanism are

1. nonnegative profits for the domestic industry—$\tau_t + p_t q_t \geq c_t q_t$;
2. nonnegative profits for foreign firms—$\tau_t^f + p_t q_t^f \geq 0$; and
3. incentive compatibility.

Several observations from Section 2 carry over to the private-information setup. As in Remarks 1-3, we can set $p_t = 0$ and $\tau_t^f = 0$ for all t, and $\tau_t = cq_t$ before the transition to zero cost. Again, it is useful to consider the government's problem before and after the transition. After the transition, there are no incentive problems and the domestic industry is as efficient as the foreign firms ($q^f = 0$), there is no need to subsidize the domestic industry ($\tau = 0$), and $q = p^{-1}(0)$; if the transition occurs at T, the government's continuation value is $\frac{S}{r} - M_T$.[9]

[8] The reward can be either a one-time payment at time t or, equivalently, a sequence of constant payments after t with its present value being M_t.

[9] Technically, we have to make sure these simplifications of policy instruments do not violate any incentive constraints. The formal analysis requires messy notation, but it is easy to see (i) setting $p_t = 0$ and $\tau_t^f = 0$ for all t does not affect the domestic industry's payoff whether it cheats or not, (ii) setting $\tau_t = cq_t$ before the transition reduces the domestic industry's payoff from cheating, thus relaxing the incentive constraint, and (iii) setting post-transition subsidy $\tau = 0$ is innocuous since the subsidies, if any, can be subsumed into the one-time reward M, so the domestic industry's continuation payoff upon transition does not change.

Import quota As in Section 2, q_t^f affects only the flow payoff at t, so the optimal import quota is a stationary function of domestic output. Equation (2) and Lemma 1 continue to hold in the private-information case. The function $q^f(q)$ is identical to the one in the public-information case since, conditional on q, there are no dynamic or incentive problems associated with choosing q^f.

Thus, the government's objective at time 0 is

$$E\left[\int_0^T e^{-rt}\left(\int_0^{q_t+q^f(q_t)} p(Q)dQ - cq_t - \Gamma(q^f(q_t))\right)dt + \int_T^\infty e^{-rt}Sdt - e^{-rT}M_T\right],$$

where the first term in parentheses is the flow payoff before the transition and T is the random transition time. Using $E\left[e^{-rT}M_T\right] = \int_0^\infty e^{-rt-\Pi_t}\pi(q_t)M_t dt$ and (4), we can rewrite the objective as

$$\frac{S}{r} - \int_0^\infty e^{-rt-\Pi_t}\left(L(q_t) + cq_t + \pi(q_t)M_t\right)dt. \tag{7}$$

So the government's problem is to maximize (7) subject to incentive compatibility. As in Section 2, we will exclude $\frac{S}{r}$ from our analysis and minimize the losses in (7).

Remark 4 *Recall that our direct mechanism has a nonnegative-profits constraint in each period for the domestic industry rather than one participation constraint at time 0. The private-information problem becomes trivial under the latter: The optimal domestic output is the public-information solution $q_t = q^{pub}, \forall t$. To see this, note that $\int_0^\infty e^{-rt-\Pi_t}\pi(q_t)M_t dt$ in (7) is the total discounted cost of rewarding the domestic industry. The government can collect this money upfront, in which case the net cost of reward is zero.*[10] *After removing this cost, the objective in (7) is the same as that in (4). Upfront payment is feasible under the time-0 participation constraint because the domestic industry's discounted value at $t = 0$ is 0. Under the period-by-period constraint, however, upfront payment is no longer feasible and the private-information problem becomes nontrivial.*

[10] Net cost of reward is also zero in Dinopoulos, Lewis, and Sappington (1995). In their two-period model of strategic trade policy, the incentive constraints do not bind and the optimal allocation under private information is the same as that under public information.

Incentive compatibility If the industry transitions at time t but decides to postpone the report of the transition until $\tilde{t} > t$, then it continues to receive subsidies from t to $\tilde{t}$. The payoff from cheating is $\int_t^{\tilde{t}} e^{-r(s-t)} c q_s ds + e^{-r(\tilde{t}-t)} M_{\tilde{t}}$. Incentive compatibility requires that

$$M_t \geq \int_t^{\tilde{t}} e^{-r(s-t)} c q_s ds + e^{-r(\tilde{t}-t)} M_{\tilde{t}}, \quad \forall t \geq 0, \forall \tilde{t} > t. \tag{8}$$

In particular, when $\tilde{t} = \infty$ the above constraint becomes

$$M_t \geq \int_t^{\infty} e^{-r(s-t)} c q_s ds, \quad \forall t \geq 0. \tag{9}$$

There is no incentive to cheat in the other direction, i.e, report a transition to zero cost without an actual transition. We demonstrate this in Lemma 7 in Appendix A.

In the following analysis, we consider a "relaxed" problem, in which we minimize the losses in (7) subject to (9). That is, we replace incentive constraints (8) by (9), a strict subset of those in (8). The following lemma shows that (9) binds for all $t \geq 0$ in the relaxed problem, which implies (8) holds for all $t < \tilde{t}$ in the original problem.

Lemma 2 (Relaxed problem) *Minimizing the losses in* (7) *subject to* (9)*, the optimal solution has*

$$M_t = \int_t^{\infty} e^{-r(s-t)} c q_s ds, \quad \forall t \geq 0. \tag{10}$$

Thus, (8) *holds as an equality, and the solution to the relaxed problem solves the original problem.*

Lemma 2 allows us to simplify the losses in (7) as

$$\begin{aligned}
& \int_0^{\infty} e^{-rt-\Pi_t} \left(L(q_t) + c q_t + \pi(q_t) M_t \right) dt \\
= & \int_0^{\infty} e^{-rt-\Pi_t} (L(q_t) + c q_t) dt + \int_0^{\infty} \pi(q_t) e^{-rt-\Pi_t} \left(\int_t^{\infty} e^{-r(s-t)} c q_s ds \right) dt \\
= & \int_0^{\infty} e^{-rt-\Pi_t} (L(q_t) + c q_t) dt + \int_0^{\infty} \left(e^{-rs} \int_0^{s} \pi(q_t) e^{-\Pi_t} dt \right) c q_s ds \\
= & \int_0^{\infty} e^{-rt-\Pi_t} (L(q_t) + c q_t) dt + \int_0^{\infty} (1 - e^{-\Pi_t}) e^{-rt} c q_t dt
\end{aligned}$$

$$= \int_0^\infty e^{-rt-\Pi_t} L(q_t)dt + \int_0^\infty e^{-rt} c q_t dt, \quad (11)$$

where the change of the order of integration in the third line follows from Fubini's theorem. Unlike equation (4), where the probability $e^{-\Pi_t}$ appears in front of both the social cost $L(q_t)$ and the production subsidy cq_t, here the probability $e^{-\Pi_t}$ is only in front of $L(q_t)$. This is because the government does not observe the transition and must offer a reward to induce truth-telling. The reward M_t equals the production subsidy to the domestic industry had it cheated after transition. So it is as if the industry receives the subsidy both before and after transition; i.e., the subsidy is *unconditional*. Thus, private information makes protection more costly.

Differentiating (11) with respect to $q_t \geq 0$, we derive the first-order condition as

$$e^{-rt-\Pi_t} L'(q_t) - \int_t^\infty e^{-rs-\Pi_s} \pi'(q_t) L(q_s) ds + e^{-rt} c \geq 0,$$

or simply

$$-e^{-\Pi_t} L'(q_t) + \left[\int_t^\infty e^{-r(s-t)-\Pi_s} L(q_s) ds \right] \pi'(q_t) \leq c, \quad (12)$$

which holds as an equality if $q_t > 0$. The right-hand side of (12) is the cost of protection: For each unit of output produced by the domestic industry, the government provides a subsidy c. On the left-hand side of (12) are the two benefits of protection: The first term is the reduced social cost of imports when q_t is higher $(-L'(q_t) = \Gamma'(q^f(q_t)))$, while the second term represents the benefit of learning by doing. Specifically, higher q_t increases the probability of transition and $\int_t^\infty e^{-r(s-t)-\Pi_s} L(q_s) ds$ is the discounted social gain associated with this event.

To determine whether the industry is worth protecting at all (i.e., the Bastable condition) under private information, consider the benefit from protection in equation (12) at $t = 0$, i.e., a temporary variation $q_0 = \epsilon > 0$ but $q_t = 0$ for all $t > 0$. The benefit as of time 0 would be the sum of the marginal social gain, $-e^0 L'(0)$, and the permanent gain due to the increase in probability of a transition to zero cost $\left[\int_0^\infty e^{-rs} L(0) ds\right] \pi'(0)$. The sum of the two benefits is exactly the same as in the public-information model (see description below equation (5)). For the benefit to exceed the cost of protection, c must be less than $\bar{c} \equiv -L'(0) + \pi'(0)\frac{L(0)}{r}$. This is the same as the Bastable condition (5) under public information.[11]

LEMMA 3 (Bastable condition) *The optimal protection satisfies $q_t = 0\ \forall t \geq 0$ if and only if $c \geq \bar{c}$.*

Hereafter, we shall impose the Bastable condition, $c < \bar{c} < \infty$, so that the optimal-protection problem is nontrivial.

A sequential problem Using (11), the government's problem can be written as an optimal-control problem with state variable Π_t and control variable q_t:

$$\mathcal{L}(\Pi) \equiv \min_{\{q_t\}_{t\geq 0}} \quad \int_0^\infty e^{-rt-\Pi_t} L(q_t)dt + \int_0^\infty e^{-rt} cq_t dt \tag{13}$$
$$\text{s.t.} \quad \frac{d\Pi_t}{dt} = \pi(q_t), \quad \Pi_0 = \Pi.$$

The state variable Π represents the accumulated learning prior to the current period. Although $\Pi_0 = 0$, we treat $\Pi_0 = \Pi$ as any nonnegative number for the rest of the paper so that we can use $[0, \infty)$ as the state space in a recursive formulation.

Differentiating the objective function (13) with respect to Π yields

$$\mathcal{L}'(\Pi) = -\int_0^\infty e^{-rt-\Pi_t} L(q_t)dt, \tag{14}$$

where $\{q_t\}_{t\geq 0}$ is the path of optimal controls in problem (13). Equation (14) implies $\mathcal{L}'(\Pi) < 0$ since it cannot be the case that the losses $L(q_t)$ are zero for all $t \geq 0$: Zero losses forever would imply the domestic output is the same before and after the transition to zero cost.[12]

In the rest of this section, we characterize the optimal solution using a recursive formulation. We show that the optimal domestic output is monotonically decreasing

[11] One might wonder why M_0, which is the reward for reporting a transition at time 0 under private information, does not appear in the Bastable condition. It implicitly does: As noted earlier, the subsidy is unconditional under private information. For the temporary variation at time 0, the difference between the subsidy under private information and that under public information is M_0. Since $M_0 = c\epsilon$ and the probability of transition at 0 is $\pi(\epsilon)$, the expected value of reward for truthful reporting is $c\epsilon\pi(\epsilon)$, a higher order infinitesimal than ϵ. So, this extra cost does not have a first-order effect and the Bastable condition remains the same under private- and public-information scenarios.

[12] If $L(q_t) = 0, \forall t$, then $q_t = p^{-1}(0), \forall t$, which clearly violates the first-order condition (12). Since $L'(q_t) = L(q_s) = 0, \forall s \geq t$, the left side of (12) is 0 and is less than the right side, but (12) needs to hold as an equality because $q_t = p^{-1}(0) > 0$.

in the duration of protection. We also derive sufficient conditions under which protection is offered initially but terminated in finite time even if the industry has not transitioned to zero cost.

3.1 A recursive formulation

The optimal-control problem has a recursive formulation with Π as the state variable. The Hamilton-Jacobi-Bellman equation is

$$r\mathcal{L}(\Pi) = \min_{q \geq 0} e^{-\Pi} L(q) + cq + \mathcal{L}'(\Pi)\pi(q). \tag{15}$$

The right-hand side of (15) is convex in q because L is convex, $\mathcal{L}' < 0$, and π is concave. Therefore, the following first-order condition for $q \geq 0$ is both necessary and sufficient for optimality:

$$-e^{-\Pi} L'(q) - \mathcal{L}'(\Pi)\pi'(q) \leq c, \tag{16}$$

with an equality if $q > 0$. Note that substituting (14) into (12) also yields (16). Lemma 4 below shows that the policy function $q(\Pi)$ defined by (16) is monotonic.

Lemma 4 (Monotonicity of domestic output) $\frac{dq}{d\Pi} < 0$ *as long as* $q(\Pi) > 0$.

The intuition for Lemma 4 can be easily seen if we remove the dynamic benefit of protection from (16) (second term on the left side). In this case, equation (16) reduces to $-e^{-\Pi} L'(q) = c$. As Π increases, the benefit of protection is small, and hence the government is less willing to protect.

Our first private-information result is that optimal protection is always less than that under public information, monotonically decreasing over time, and disappears in the long run. See Figure 1 for a numerical example.

Theorem 2 (Decreasing protection under private information) *The optimal* $\{q_t\}_{t=0}^{\infty}$ *satisfies the following:*

1. $q_t \equiv q(\Pi_t)$ *is decreasing over time,* $\lim_{t\to\infty} q_t = 0$*, and* $\lim_{t\to\infty} \pi(q_t) = 0$.

2. $q_0 \in (q^{stat}, q^{pub})$.

A direct consequence of Lemma 4 is that the optimal domestic output is decreasing over time because Π_t is increasing over time due to learning by doing. Declining q_t (and M_t) ensures that the industry does not postpone report of transition. The intuition for $q_0 \in (q^{stat}, q^{pub})$ is as follows: At $t = 0$, (16) becomes $-L'(q_0) - \mathcal{L}'(0)\pi'(q_0) = c$, so q_0 is an increasing function of $-\mathcal{L}'(0)$. If $-\mathcal{L}'(0)$ were 0, then q_0 collapses to q^{stat} because $-L'(q_0) = p(q_0 + q^f(q_0)) = c$ is identical to condition (3) for the static optimization. On the other hand, q_t decreasing over time and $q_0 < q^{pub}$ imply $q_t < q^{pub}, \forall t$. That is, the industry gets less protection under private information *at all times*. This is not surprising since the cost of protection under private information is more than that under public information (see (11) and (4)).

Our next result shows that an eventual transition is not guaranteed under the optimal protection policy. That is, the protection policy will fail the Mill test with positive probability.

Theorem 3 (Mill test) $\lim_{t\to\infty} e^{-\Pi_t} = c/\bar{c} < 1$.

Under private information, the probability that the domestic industry would eventually transition to zero cost is positive, but it could be less than 1. That is, the protection policy passes the Mill test with positive probability but cannot pass it with certainty. So, the protection policy cannot guarantee that the industry will be able to compete internationally. Recall that under public information, the optimal domestic output is stationary, so the arrival rate of transition is constant $\pi(q^{pub}) > 0$ and the industry will eventually transition to zero cost with probability 1.

Our third result provides sufficient conditions for the optimal protection to be temporary. That is, the optimal policy lets the foreign competitors take over the entire market forever after a threshold number of periods even if the domestic industry's cost remains high.

Theorem 4 (Temporary protection)

1. *If either $L''(0) > 0$ or $\pi''(0) < 0$, then $q_t > 0$ for all t.*

2. *If there exists $\epsilon > 0$ such that $L''(q) = 0$ and $\pi''(q) = 0$ for all $q \in (0, \epsilon)$, then q_t reaches 0 in finite time and stays there.*

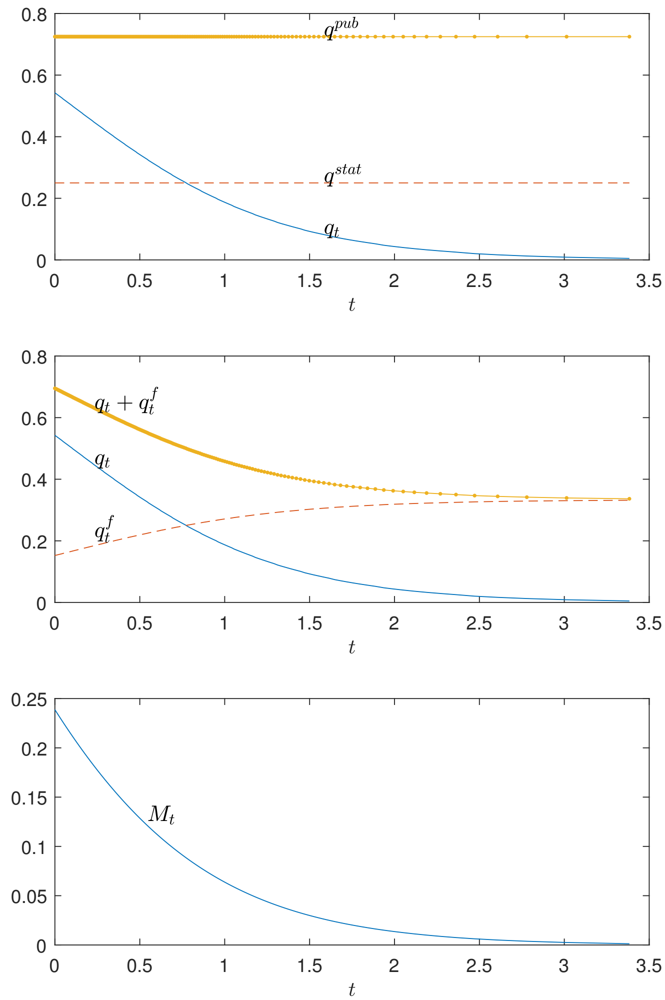

Figure 1: The top panel shows the optimal time path of domestic output q_t. The middle panel adds the optimal time path of imports q_t^f and total $q_t + q_t^f$. The bottom panel plots the reward to the domestic industry as a function of its reported transition time. In this numerical example, $r = 1$, $c = 0.5$, $\pi(q) = 2q$, $\Gamma(q^f) = (q^f)^2$, and $p(Q) = 1 - Q$. Then, $q^f(q) = \frac{1-q}{3}$, $p^{-1}(0) = 1$, and $S = \int_0^1 p(Q)dQ = 1/2$.

When both $L(q)$ and $\pi(q)$ are linear in q for small $q \in (0, \epsilon)$, they will enter the linear range eventually since q is decreasing over time. Part 2 of Theorem 4 says that local linearity implies the bang-bang property, so q will be set to zero in finite time. (See linear example in Section 6.)

3.2 Implementation

The optimal allocation in the previous section can be implemented through a one-time cash transfer to the domestic industry at $t = 0$ and a consumption tax. Recall that the socially optimal outputs are $\{(q_t, q^f(q_t))\}_{t \geq 0}$ before transition and $(p^{-1}(0), 0)$ afterward. The implementation scheme is as follows:

1. The government provides a fund $M_0 \equiv \int_0^\infty e^{-rt} c q_t dt$ to the domestic industry at time 0 and mandates that it choose output from interval $[q_t, p^{-1}(0)]$, $\forall t \geq 0$.

2. At any time t, the domestic industry produces $\tilde{q}_t \in [q_t, p^{-1}(0)]$ and the government observes $\tilde{q}_t$.

3. Then the government sets consumption tax $\tau_{ct} \equiv p(\tilde{q}_t + q^f(\tilde{q}_t))$ per unit and import quota $q^f(\tilde{q}_t)$. The tax revenue is lump-sum rebated to the consumers.

The implementation is simple: The government neither collects the report of privately observed transition nor offers subsidies or rewards to the domestic industry except the initial M_0.

THEOREM 5 (Implementation) *Facing the constraint $\tilde{q}_t \in [q_t, p^{-1}(0)]$, it is optimal for the domestic industry to replicate the allocation in the direct mechanism, i.e., produce q_t before the transition and $p^{-1}(0)$ after the transition.*

To understand our implementation, four features are worth noting. First, in the presence of positive imports the competitive market price is 0. The total quantity demanded is given by the inverse demand curve $p(Q)$ evaluated at "price" τ_c. At time t, τ_{ct} is set to $p(\tilde{q}_t + q^f(\tilde{q}_t))$, so clearly $Q_t = \tilde{q}_t + q^f(\tilde{q}_t)$. The domestic industry supplies $\tilde{q}_t$ and the foreign firms supply $q^f(\tilde{q}_t)$.

Second, before transition time T, the domestic industry's asset holding, $B_t \geq 0$, evolves as a function of domestic output $\tilde{q}_t$:

$$\frac{dB_t}{dt} = rB_t - c\tilde{q}_t, \quad B_0 = M_0. \tag{17}$$

For the domestic industry, increasing $\tilde{q}_t$ (i) depletes the fund at t and (ii) reduces the odds of depleting the fund in the future due to learning by doing. The government has already internalized the benefit of learning in setting q_t, so the domestic industry cannot gain by producing more than q_t. However, the domestic industry does not take into account the reduced social cost of imports, which is an externality internalized only by the government. Therefore, the domestic industry prefers an output below the socially optimal q_t. The constraint $\tilde{q}_t \in [q_t, p^{-1}(0)]$ ensures that the industry's choice is q_t, which is verified by Theorem 5.

Third, one can easily verify that M_t defined in (10) satisfies the budget constraint (17) when $\tilde{q}_t = q_t$, meaning that the industry's asset holding B_t is equal to M_t. If the transition arrives at T, then M_T can be interpreted as a reward from the government.

Fourth, after the transition at T, the domestic industry is indifferent to any output in the interval $[q_t, p^{-1}(0)]$ and does not gain by deviating from the socially optimal $p^{-1}(0)$. If the domestic industry produces $p^{-1}(0)$, then the consumption tax and imports are both 0, and the social welfare is maximized.

4 Time consistency

The optimal protection policy under private information is not time consistent. To see this, consider the optimal path where the domestic industry reports high cost at t; as of time 0, this event occurs with probability $e^{-\Pi_t}$. Suppose a new government arrives at t and replaces the old government. The new government faces a mechanism-design problem that is identical to the problem at time 0 because the only relevant information for the new government at t is that the domestic industry's cost is high. The probability $e^{-\Pi_t}$, based on the history of the optimal path, does not affect the new government's problem, because the new government arrives after the high-cost event is realized. Consequently, a new government will reset the time-t protection to q_0, rather than follow the old path.

We have time-inconsistency because government's preferences are dynamically inconsistent under private information. Let time-0 government's losses be $\mathbb{L}(0)$, and let time-t government's losses be $\mathbb{L}(t)$.

$$\begin{aligned}
\mathbb{L}(0) &= \int_0^\infty e^{-rs-\Pi_s} L(q_s) ds + \int_0^\infty e^{-rs} c q_s ds, \\
\mathbb{L}(t) &= \underbrace{\int_t^\infty e^{-r(s-t)-(\Pi_s-\Pi_t)} L(q_s) ds}_{A} + \underbrace{\int_t^\infty e^{-r(s-t)} c q_s ds}_{B}.
\end{aligned}$$

So,

$$\begin{aligned}
\mathbb{L}(0) &= \int_0^\infty e^{-rs-\Pi_s} L(q_s) ds + \int_0^\infty e^{-rs} c q_s ds, \\
&= \int_0^t e^{-rs-\Pi_s} L(q_s) ds + \int_0^t e^{-rs} c q_s ds + e^{-rt-\Pi_t} \left(A + e^{\Pi_t} B \right).
\end{aligned}$$

Both governments want to minimize A and B, but setting $A = 0$ and $B = 0$ simultaneously is not feasible. The tradeoff between A and B depends on their respective weights in the objective functions. While the time-t government puts equal weights on A and B, the time-0 government puts more weight on B than on A; i.e., their preferences are not dynamically consistent.[13] The time-0 government is less willing to tolerate the loss of B than the time-t government because promised subsidies after t have negative effects on incentives before t, which have to be taken into account by the time-0 government, but not by the time-t government. Consequently, the time-t government's optimal domestic output $\{q_s\}_{s \geq t}$ is greater than the time-0 government's output for $s \geq t$. That is, the time-t government would offer more protection and not continue the path of the time-0 government.

Under public information,

$$\begin{aligned}
\mathbb{L}(0) &= \int_0^\infty e^{-rs-\Pi_s} \left(L(q_s) + c q_s \right) ds, \\
\mathbb{L}(t) &= \int_t^\infty e^{-r(s-t)-(\Pi_s-\Pi_t)} \left(L(q_s) + c q_s \right) ds,
\end{aligned}$$

[13] This is similar to hyperbolic discounting as in Laibson (1997), for instance.

so $\mathbb{L}(0) = \int_0^t e^{-rs-\Pi_s}(L(q_s) + cq_s)\, ds + e^{-rt-\Pi_t}\mathbb{L}(t)$. Thus, both time-0 and time-t governments want to minimize $\mathbb{L}(t)$ and their preferences are dynamically consistent.

One way to obtain a time-consistent policy under private information is to impose stationarity: Restrict the direct mechanism to choose $q_t \equiv q, \forall t$ for some q. Matsuyama (1990) adopts a similar method in the context of a repeated public-information game. The domestic output that minimizes the government's loss under stationarity is given by

$$q^* \in \arg\min_q \int_0^\infty e^{-rt-\Pi_t} L(q)dt + c\int_0^\infty e^{-rt} q dt = \frac{L(q)}{r + \pi(q)} + \frac{c}{r}q. \tag{18}$$

The time-consistent policy, clearly, is permanent: The domestic industry is protected as long as its reported cost remains high. Since domestic output is constant, the reward at the time of transition, given by the incentive compatibility constraint (10), is also constant. A constant reward is incentive compatible due to a positive rate of time preference in our model: If the transition occurs at t, then reporting the truth and receiving the reward at t weakly dominates postponing the report.

Unsurprisingly, the welfare under the time-consistent policy q^* is less than that under the optimal $\{q_t\}_{t=0}^\infty$ in Theorem 2 since the time-consistent policy is feasible in problem (13). The two time paths of protection, however, cannot be ranked. In fact, q^* is initially lower but eventually higher than q_t.

Theorem 6 (Time-consistent protection) $\lim_{t\to\infty} q_t = 0 < q^* < q_0$.

Since q^* maximizes the same objective as $\{q_t\}_{t=0}^\infty$, an important criterion for the path of q^* is that it should stay close to $\{q_t\}_{t=0}^\infty$. Since one path is constant and the other is decreasing over time, this is achieved by letting the two time paths cross at some $t > 0$. Choosing either $q^* = 0$ or $q^* \geq q_0$ will push them further apart.

Finally, it is easy to see that the objective (18) puts a higher weight on the subsidy than (6) under public information. Higher protection cost in (18) implies that the time-consistent policy under private information offers less protection, i.e., $q^* < q^{pub}$.

Note that, similar to the public-information setup, constancy of domestic output in the time-consistent policy implies that the domestic industry will eventually be able to compete internationally with probability 1. However, passing the Mill test with certainty is not optimal.

Another approach is to study the time-consistent mechanisms that may not be stationary. This is challenging because we cannot apply the revelation principle, which holds only under full commitment. We conjecture that the mechanism will let time-0 government send noisy reports to future governments. Because a future government cannot be sure that an industry reporting high cost does indeed have high cost, future government's belief about the industry type becomes a state variable. The reports from the time-0 government will make future governments believe in a higher probability of lying by the industry. This belief mitigates future governments' incentives to increase protection. In other words, by introducing deteriorating beliefs, the time-0 government can make future governments' objective align with a declining path $\{q_t\}_{t\geq 0}$.

5 Which industries to protect?

Our analysis in the previous sections was about protecting one industry optimally when the government has access to unlimited resources. Suppose the government receives requests from multiple industries for protection. With unlimited resources, the government would protect all industries that satisfy the Bastable condition and the optimal protection of one industry would not affect the protection of other industries. With limited resources, which industries should be protected? In this section, we assume that the government's resources are limited by F. All other features of our environment in Section 3 are retained in this section.

First, consider a single-industry problem under limited resources: The total subsidy and reward cannot exceed F. Thus,

$$\begin{aligned} F &\geq \int_0^\infty e^{-rt-\Pi_t}\left(cq_t + \pi(q_t)M_t\right)dt \\ &= \int_0^\infty e^{-rt} cq_t dt. \end{aligned} \tag{19}$$

The government wants to minimize the losses in (11), subject to (19). The Lagrangian of this problem is

$$\int_0^\infty e^{-rt-\Pi_t}L(q_t)dt + \int_0^\infty e^{-rt}cq_t dt + \theta\left(\int_0^\infty e^{-rt}cq_t dt - F\right), \tag{20}$$

where $\theta \geq 0$ is the Lagrange multiplier on constraint (19). Problem (20) is equivalent to an otherwise identical private-information problem except the domestic industry's cost is $(1+\theta)c$ instead of c. That is, a limited-resource model can be transformed into an unlimited-resource model with a magnified cost. Under limited resources, the Bastable condition is $(1+\theta)c < \bar{c}$, which could be violated even if $c < \bar{c}$. Note that $\bar{c}$ is the same in the two problems.

Second, consider a problem with multiple industries, indexed by $i \in \{1, 2, ..., I\}$, and limited resources. (An industry in our model is defined by the 4-tuple: c, $\pi(\cdot)$, $\Gamma(\cdot)$, and $p(\cdot)$.) The government's budget constraint becomes

$$F \geq \int_0^\infty e^{-rt} \left(\sum_{i=1}^{I} c^i q_t^i \right) dt, \tag{21}$$

where c^i and q_t^i are, respectively, the cost and domestic output in industry i. Conditional on the multiplier θ on (21), the optimal protection of each industry is still captured by problem (20) but with industry-specific 4-tuple. If F decreases (or there are more industries competing for F), then θ will increase, which is isomorphic to magnifying the cost in each industry by the same proportion.

The multiplier θ helps the government decide which industries should be protected. For industry i, the government can use the magnified cost $(1+\theta)c^i$ and rule it out from protection if its magnified cost exceeds $\bar{c}^i$. That is, industry i satisfies the Bastable condition if

$$(1+\theta)c^i < \bar{c}^i. \tag{22}$$

Finally, since problem (20) is identical to our benchmark model in Section 3, all the qualitative properties in Section 3 continue to hold. In particular, the optimal amount of protection will decline over time in every protected industry.

5.1 Determining the "magnifier" θ

We use a bisection method to solve for θ. The algorithm is as follows:

- Initialization: Set $\overline{\theta}$ to a large number and $\underline{\theta} = 0$.
- Choose a stopping criterion ϵ.

- Step 1. Set $\theta = (\underline{\theta} + \overline{\theta})/2$. For $i \in \{1, 2, ..., I\}$,
  - if industry i satisfies the Bastable condition (22), then solve for the optimal protection policy $\{q_t^i\}_{t=0}^{\infty}$ for i with its production cost magnified from c^i to $(1+\theta)c^i$, and with its industry-specific characteristics;
  - otherwise, set $q_t^i = 0$ for all $t \geq 0$.
- Step 2.
  - If $F > \int_0^\infty e^{-rt} \left(\sum_{i=1}^I c^i q_t^i \right) dt + \epsilon$, then reset $\overline{\theta} = \theta$ and go to Step 1;
  - if $F < \int_0^\infty e^{-rt} \left(\sum_{i=1}^I c^i q_t^i \right) dt - \epsilon$, then reset $\underline{\theta} = \theta$ and go to Step 1;
  - otherwise, stop and report solution at $\theta = (\underline{\theta} + \overline{\theta})/2$.

6 A linear example

The demand for the product is inelastic: A unit measure of agents wants to consume one unit of the product each. The social cost of imports is linear: $\Gamma(q^f) \equiv \gamma \cdot q^f$, $\gamma > 0$. The Poisson process is also linear: $\pi(q) \equiv \pi \cdot q$, $\pi > 0$. For the industry to be protected, the parameters have to satisfy the Bastable condition: $c < \bar{c} \equiv \frac{\pi}{r}\gamma + \gamma$.

With inelastic demand, the optimal protection policy implies $q_t + q_t^f = 1$ for all t. After the transition, it is optimal to set $q = 1$ forever since there is no reason to incur the social cost of imports. Recall that S is the flow of consumer surplus (also the social surplus) after the transition at T and the government's payoff is $\int_T^\infty e^{-rt} S dt - e^{-rT} M_T$. Prior to the transition at T, the government's payoff is $\int_0^T e^{-rt}(S - \gamma \cdot (1 - q_t) - cq_t)dt$. The government's objective at time 0 is to maximize

$$E\left[\int_0^T e^{-rt} \left(S - \gamma \cdot (1 - q_t) - cq_t\right) dt + e^{-rT} \left(\frac{S}{r} - M_T\right)\right], \tag{23}$$

subject to incentive compatibility (10).

As is typically the case in linear continuous-time optimal-control problems, the optimal path for q_t is a step function.

Theorem 7 (Bang-bang protection) *The optimal path of q_t is given by*

$$q_t = \begin{cases} 1, & \text{if } t \leq \bar{t}; \\ 0, & \text{if } t > \bar{t}, \end{cases} \tag{24}$$

where $\bar{t}$ is the maximal duration of protection for the domestic industry and solves

$$e^{-\pi\bar{t}}\gamma + \pi\frac{\gamma}{r}e^{-\pi\bar{t}} = c. \tag{25}$$

The reward for a transition at time t, M_t, satisfies

$$M_t = \begin{cases} \frac{c}{r}(1 - e^{r(t-\bar{t})}), & \text{if } t \leq \bar{t}; \\ 0, & \text{if } t > \bar{t}. \end{cases} \tag{26}$$

Note that $\bar{t}$, determined at time 0, is finite and the protection ceases at $\bar{t}$ even if the industry has not transitioned to zero cost by that time. That is, $q_t = 0$ and $q_t^f = 1$ for all $t \geq \bar{t}$. With $q = 1$ before the transition, the probability that the industry would transition to zero cost by time t is $e^{-\pi t}$, so at $\bar{t}$ there is a positive probability the industry's cost remains high. In other words, there is a positive probability the industry would fail the Mill test.

6.1 Multiple industries and limited resources

We numerically illustrate the role of limited resources using the above linear example with five industries. We set $r = 0.05$. The demand for each industry's product is the same: one unit, inelastic. The rest of the parameters are in Table 1.

Industry	c_i	γ_i	π_i
1	0.1	0.6	0.6
2	0.2	0.6	0.6
3	0.1	0.2	0.6
4	0.1	0.6	0.1
5	0.2	0.4	0.1

Table 1: Parameters

The parameters in Table 1 are chosen so that $\frac{c^i}{\bar{c}^i}$ is increasing in i. Industry i is ranked higher than $i+1$ in the sense that if $i+1$ receives protection then i will also receive protection; see (22) for the Bastable condition. Conditional on protection, industry i has a higher probability of passing the Mill test than industry $i+1$.

Tables 2-4 show that more industries are protected as the resources F increase. With few resources ($F = 0.10$), only industry 1 is protected: The multiplier θ on the resource constraint is high and the Bastable condition $\frac{(1+\theta)c^i}{\bar{c}^i} < 1$ is satisfied only by $i = 1$. With ample resources ($F = 3.00$), all five industries are protected.

7 Conclusion

We study infant industry protection in a dynamic model where initially the industry cannot compete with foreign firms that possess a superior technology. The industry can stochastically reduce its cost through learning by doing, but the transition to low cost is private information. We use a mechanism-design approach and establish that the optimal protection declines over time and can be implemented with minimal information requirements. When the resources to protect multiple industries are limited, we deliver a simple approach to choose which industries to protect. If the transition to low cost is public information, then we show that it is optimal to offer "permanent" protection: Subsidize the domestic industry until it can compete with foreign firms. Furthermore, the optimal policy under public information is time consistent, but under private information it is not.

A lesson from our model is that information regarding the evolution of an infant industry is critical for the optimal protection policy. The public-information case in our model can be viewed as one where all information regarding the industry can be freely obtained, while the private-information case is one where obtaining information is infinitely costly. An intermediate case, where the government can verify the industry's information by incurring a finite cost, is worth studying. We conjecture that the optimal protection policy would involve periodic verification.

Our paper is the first attempt to study the dynamic incentive problems involved in infant industry protection. We have focused on the incentive problem that the government cannot observe the infant industry's transition, but other incentive problems might also be plausible. For example, the government may provide funds for the domestic industry's R&D activities but cannot monitor how the industry uses these funds. The government may lack information about the domestic industry's R&D capability, giving the latter an incentive to misreport and receive more protection.

	$F = 0.10$	$F = 0.20$	$F = 0.40$	$F = 1.00$	$F = 3.00$
Industry	$\theta = 41.15$	$\theta = 31.57$	$\theta = 21.50$	$\theta = 12.36$	$\theta = 3.32$
1	0.54	0.42	0.29	0.17	0.06
2	1.08	0.84	0.58	0.34	0.11
3	1.62	1.25	0.87	0.51	0.17
4	2.34	1.81	1.25	0.74	0.24
5	7.02	5.43	3.75	2.23	0.72

Table 2: Ratio $\frac{(1+\theta)c^i}{\bar{c}^i}$

	$F = 0.10$	$F = 0.20$	$F = 0.40$	$F = 1.00$	$F = 3.00$
Industry	$\theta = 41.15$	$\theta = 31.57$	$\theta = 21.50$	$\theta = 12.36$	$\theta = 3.32$
1	1.03	1.46	2.07	2.94	4.82
2	0.00	0.30	0.92	1.79	3.67
3	0.00	0.00	0.24	1.11	2.99
4	0.00	0.00	0.00	2.98	14.26
5	0.00	0.00	0.00	0.00	3.28

Table 3: Duration $\bar{t}^i$

	$F = 0.10$	$F = 0.20$	$F = 0.40$	$F = 1.00$	$F = 3.00$
Industry	$\theta = 41.15$	$\theta = 31.57$	$\theta = 21.50$	$\theta = 12.36$	$\theta = 3.32$
1	0.10	0.14	0.20	0.27	0.43
2	0.00	0.06	0.18	0.34	0.67
3	0.00	0.00	0.02	0.11	0.28
4	0.00	0.00	0.00	0.28	1.02
5	0.00	0.00	0.00	0.00	0.60

Table 4: Allocation of resources

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Appendix A

Proof of Lemma 1: Totally differentiating (2) yields

$$\frac{dq^f}{dq} = \frac{p'}{\Gamma'' - p'} < 0, \quad \frac{d(q^f + q)}{dq} = \frac{\Gamma''}{\Gamma'' - p'} > 0.$$

■

Proof of Theorem 1:

1. We show that $\frac{L(q)+cq}{r+\pi(q)}$ has a unique minimum. Its derivative with respect to q is

   $$\frac{(L'(q)+c)(r+\pi(q)) - (L(q)+cq)\pi'(q)}{(r+\pi(q))^2}.$$

   The derivative of the above numerator is

   $$L''(q)(r+\pi(q)) - (L(q)+cq)\pi''(q) > 0,$$

   which means the numerator $(L'(q)+c)(r+\pi(q)) - (L(q)+cq)\pi'(q)$ is strictly increasing in q. Moreover, it is positive at $q = p^{-1}(0)$:

   $$\begin{aligned} & (L'(q)+c)(r+\pi(q)) - (L(q)+cq)\pi'(q)|_{q=p^{-1}(0)} \\ = \; & c(r+\pi(q) - \pi'(q)q) > 0. \end{aligned}$$

   Therefore, either $(L'(q)+c)(r+\pi(q)) - (L(q)+cq)\pi'(q)$ is always positive, or it is first negative and then positive. The optimal q^{pub} is 0 in the first case and the solution to $(L'(q)+c)(r+\pi(q)) - (L(q)+cq)\pi'(q) = 0$ in the second case. In both cases, the optimal q^{pub} is unique.

2. That $q^{pub} > 0$ is equivalent to the condition that

   $$\frac{(L'(q)+c)(r+\pi(q)) - (L(q)+cq)\pi'(q)}{(r+\pi(q))^2}|_{q=0} < 0.$$

The above condition simplifies to

$$(L'(0) + c)r - L(0)\pi'(0) < 0,$$

or

$$c < \bar{c}, \text{ where } \bar{c} \equiv \frac{\pi'(0)}{r}L(0) - L'(0).$$

3. If $q^{stat} > 0$, we show $q^{pub} > q^{stat}$. The first-order condition for q^{stat} is $c = U'(q^{stat}) = -L'(q^{stat})$, which implies

$$(L'(q^{stat}) + c)(r + \pi(q^{stat})) - (L(q^{stat}) + cq^{stat})\pi'(q^{stat}) < 0.$$

Therefore, $q^{pub} > q^{stat}$.

■

PROOF OF LEMMA 2: First, consider a relaxed problem where the government minimizes the losses in (7) subject to (9).

$$\begin{aligned} \min \quad & \int_0^\infty e^{-rt-\Pi_t} \left(L(q_t) + cq_t + \pi(q_t)M_t\right) dt, \\ s.t. \quad & M_t \geq \int_t^\infty e^{-r(s-t)} cq_s ds. \end{aligned}$$

Since the above objective function is increasing in M_t, it is obvious that (9) binds in the optimal solution.

Second, if $M_t = \int_t^\infty e^{-r(s-t)} cq_s ds$ for all t, then the incentive constraint (8) holds as equality for all t and $\tilde{t}$. Therefore, the solution to the relaxed problem is indeed incentive compatible. ■

PROOF OF LEMMA 3: If $c \geq \bar{c}$, then $q_t = 0, \forall t \geq 0$ is optimal because it satisfies the first-order condition in (12). To verify (12), note

$$-e^{-\Pi_t} L'(q_t) + \left[\int_t^\infty e^{-r(s-t)-\Pi_s} L(q_s) ds\right] \pi'(q_t)$$

$$= \quad -L'(0) + \left[\int_t^\infty e^{-r(s-t)} L(0) ds\right] \pi'(0) = \bar{c} \le c.$$

If $c < \bar{c}$, then consider a stationary plan where $q_t = q, \forall t$ for some $q \ge 0$. Then the government's cost function is

$$\mathcal{L}_q = \int_0^\infty e^{-rt-\Pi_t} L(q_t) dt + c \int_0^\infty e^{-rt} q_t dt = \frac{L(q)}{r + \pi(q)} + \frac{c}{r} q.$$

We have

$$\begin{aligned} \frac{d\mathcal{L}_q}{dq}|_{q=0} &= \frac{c}{r} + \frac{L'(q)(r+\pi(q)) - L(q)\pi'(q)}{(r+\pi(q))^2}|_{q=0} \\ &= \frac{c}{r} - \left(\frac{\pi'(0)}{r^2} L(0) - \frac{L'(0)}{r}\right) = \frac{c - \bar{c}}{r} < 0. \end{aligned}$$

Therefore $\mathcal{L}_q < \mathcal{L}_0$ for small $q > 0$, implying that $q_t = 0, \forall t$ is suboptimal. ■

PROOF OF LEMMA 4: First, the HJB equation (15) is

$$r\mathcal{L}(\Pi) = e^{-\Pi} L(\hat{q}) + c\hat{q} + \mathcal{L}'(\Pi)\pi(\hat{q}), \tag{27}$$

where $\hat{q}$ is the optimal policy. Applying the Envelope theorem to (27), we have

$$r\mathcal{L}'(\Pi) = -e^{-\Pi} L(\hat{q}) + \mathcal{L}''(\Pi)\pi(\hat{q}). \tag{28}$$

Summing up (27) and (28), we have, for $\hat{q} > 0$,

$$\mathcal{L}'(\Pi) + \mathcal{L}''(\Pi) = \frac{r(\mathcal{L}(\Pi) + \mathcal{L}'(\Pi)) - c\hat{q}}{\pi(\hat{q})}. \tag{29}$$

Second, totally differentiating (16) with respect to Π and $\hat{q} > 0$ yields

$$\begin{aligned} \frac{d\hat{q}}{d\Pi} &= -\frac{-e^{-\Pi} L'(\hat{q}) + \mathcal{L}''(\Pi)\pi'(\hat{q})}{e^{-\Pi} L''(\hat{q}) + \mathcal{L}'(\Pi)\pi''(\hat{q})} \\ &= -\frac{\mathcal{L}'(\Pi)\pi'(\hat{q}) + c + \mathcal{L}''(\Pi)\pi'(\hat{q})}{e^{-\Pi} L''(\hat{q}) + \mathcal{L}'(\Pi)\pi''(\hat{q})} \\ &= -\frac{\frac{(\mathcal{L}(\Pi)+\mathcal{L}'(\Pi))r\pi'(\hat{q}) + (\pi(\hat{q}) - \pi'(\hat{q})\hat{q})c}{\pi(\hat{q})}}{e^{-\Pi} L''(\hat{q}) + \mathcal{L}'(\Pi)\pi''(\hat{q})} < 0, \end{aligned} \tag{30}$$

where the second equality follows from (16), the third equality from (29), and the last inequality follows from the assumption that $\hat{q} > 0$ and $\pi(\hat{q}) > \pi'(\hat{q})\hat{q}$. ■

LEMMA 5 $-\mathcal{L}(\Pi) \leq \mathcal{L}'(\Pi) < 0$, *where the equality holds if and only if* $q_t = 0$ *for all* $t \geq 0$.

PROOF: That $-\mathcal{L}(\Pi) \leq \mathcal{L}'(\Pi)$ holds because $\mathcal{L}'(\Pi) = -\int_0^\infty e^{-rt}e^{-\Pi_t}L(q_t)dt$ and

$$-\mathcal{L}(\Pi) = -\int_0^\infty e^{-rt}(e^{-\Pi_t}L(q_t) + cq_t)dt \leq -\int_0^\infty e^{-rt}e^{-\Pi_t}L(q_t)dt.$$

Clearly, the equality holds if and only if $\int_0^\infty e^{-rt}q_t dt = 0$, or $q_t = 0, \forall t \geq 0$. ■

PROOF OF THEOREM 2:

1. By contradiction, suppose $\underline{q} \equiv \lim_{t\to\infty} q_t > 0$ and $\underline{\pi} \equiv \lim_{t\to\infty} \pi(q_t) > 0$. Then $\frac{d\Pi_t}{dt} = \pi(q_t) > \underline{\pi} > 0$ and $\lim_{t\to\infty} \Pi_t = \infty$. Taking limit $\Pi \to \infty$ in (16) yields

$$0L'(\underline{q}) - 0\pi'(\underline{q}) = c,$$

   which cannot hold as long as $\underline{q} > 0$.

2. First, we show $q_0 > 0$. Since $q_t = 0, \forall t \geq 0$ is suboptimal under the Bastable condition, the optimal path satisfies $q_{t^*} > 0$ at least for some t^*. As part 1 shows, because Π_t is increasing over time and $\frac{dq}{d\Pi} < 0$ (Lemma 4), q_t is decreasing over time, which implies $q_0 \geq q_{t^*} > 0$.

   Second, we show $q_0 > q^{stat}$. Because $q_0 > 0$, the first-order condition (16) for q_0 becomes $-L'(q_0) - \mathcal{L}'(0)\pi'(q_0) = c$. It follows from $\mathcal{L}'(0) < 0$ and the first-order condition (3) for q^{stat} that $q_0 > q^{stat}$.

   Third, to show $q_0 < q^{pub}$, we first show

$$\mathcal{L}'(0) > -\frac{cq^{pub} + L(q^{pub})}{r + \pi(q^{pub})}. \tag{31}$$

By contradiction, suppose $\mathcal{L}'(0) \leq -\frac{cq^{pub}+L(q^{pub})}{r+\pi(q^{pub})}$. Then

$$\begin{aligned} r\mathcal{L}(0) &= \min_q L(q) + cq + \mathcal{L}'(0)\pi(q) \\ &\leq \min_q L(q) + cq - \frac{cq^{pub}+L(q^{pub})}{r+\pi(q^{pub})}\pi(q) \\ &= L(q^{pub}) + cq^{pub} - \frac{cq^{pub}+L(q^{pub})}{r+\pi(q^{pub})}\pi(q^{pub}) = r\frac{cq^{pub}+L(q^{pub})}{r+\pi(q^{pub})}. \end{aligned}$$

This contradicts the fact that private-information cost $\mathcal{L}(0)$ must be higher than the public-information cost. It follow from $q_0 \in \arg\max_q L(q) + cq + \mathcal{L}'(0)\pi(q)$, $q^{pub} \in \arg\max_q L(q) + cq - \frac{cq^{pub}+L(q^{pub})}{r+\pi(q^{pub})}\pi(q)$, and (31) that $q_0 < q^{pub}$.

■

PROOF OF Theorem 3:

Define $\bar{\Pi} \equiv \log(\frac{\bar{c}}{c})$. On $[\bar{\Pi}, \infty)$, we can verify that $\mathcal{L}(\Pi) = \frac{e^{-\Pi}L(0)}{r}$ and $q(\Pi) = 0 = \pi(q(\Pi))$ solve the HJB equation. So Π_t cannot exceed $\bar{\Pi}$, and therefore $\Pi_\infty \equiv \lim_{t\to\infty}\Pi_t \leq \bar{\Pi}$. Since Theorem 2 shows $\lim_{t\to\infty} q_t = \lim_{t\to\infty} q(\Pi_t) = 0$, the monotonicity of $q(\Pi_t)$ implies $q(\Pi) = 0$ for all $\Pi \geq \Pi_\infty$, therefore, $\mathcal{L}(\Pi) = \frac{e^{-\Pi}L(0)}{r}$ on $[\Pi_\infty, \infty)$. Therefore, condition (16) at Π_∞ implies

$$\begin{aligned} c \geq -e^{-\Pi_\infty}L'(0) - \mathcal{L}'(\Pi_\infty)\pi'(0) &= -e^{-\Pi_\infty}L'(0) + \frac{e^{-\Pi_\infty}L(0)\pi'(0)}{r} \\ &= e^{-\Pi_\infty}\bar{c} \geq e^{-\bar{\Pi}}\bar{c} = c, \end{aligned}$$

which implies $\Pi_\infty = \bar{\Pi}$. ■

PROOF OF Theorem 4:

1. To show $q_t > 0, \forall t$, suppose by contradiction q_t reaches 0 for the first time at some $t^* > 0$. It follows from (30) that

$$-\frac{dq}{d\Pi}|_{\Pi=\Pi_{t^*}} = H \equiv \frac{-e^{-\Pi_{t^*}}L'(0) + \mathcal{L}''(\Pi_{t^*})\pi'(0)}{e^{-\Pi_{t^*}}L''(0) + \mathcal{L}'(\Pi_{t^*})\pi''(0)},$$

where H is finite under the assumption of either $L''(0) > 0$ or $\pi''(0) < 0$. Pick a small $\epsilon > 0$, such that for all $t \in (t^* - \epsilon, t^*)$, q_t is close to 0, Π_t is close to Π_{t^*}, and

$$\pi(q_t) < 2\pi'(0)q_t, \quad -\frac{dq}{d\Pi}|_{\Pi=\Pi_t} < 2H.$$

Therefore, $-\frac{dq_t}{dt} = -\frac{dq_t}{d\Pi}\frac{d\Pi}{dt} = -\frac{dq_t}{d\Pi}\pi(q_t) < 4H\pi'(0)q_t$. It follows from Gronwall's inequality that $q_{t^*} \geq q_{t^*-\epsilon}e^{-4H\pi'(0)\epsilon}$, which contradicts the assumption that $q_{t^*} = 0$.

2. We show that q_t reaches 0 in finite time and stays there afterward. Since $\lim_{t\to\infty} q_t = 0$, pick a t such that $q_t \leq \epsilon$. First, we show $\mathcal{L}(\Pi_t) = \frac{e^{-\Pi_t}L(0)}{r}$. The first-order condition for $q_t \geq 0$ is $e^{-\Pi_t}L'(q_t) + c + \mathcal{L}'(\Pi_t)\pi'(q_t) \geq 0$ (becoming an equality if $q_t > 0$). Since $L'(q_t) = L'(0)$ and $\pi'(q_t) = \pi'(0)$, both 0 and q_t are optimal solutions. Therefore, the HJB equation (15) becomes

$$r\mathcal{L}(\Pi_t) = e^{-\Pi_t}L(0) + c0 + \mathcal{L}'(\Pi_t)\pi(0) = e^{-\Pi_t}L(0).$$

Second, we show $\mathcal{L}(\Pi) = \frac{e^{-\Pi}L(0)}{r}$ for all $\Pi \geq \Pi_t$. On the one hand, $\mathcal{L}(\Pi) \geq \frac{e^{-\Pi}L(0)}{r}$ because $-\mathcal{L}(\Pi) \leq \mathcal{L}'(\Pi)$ in Lemma 5 implies that $\mathcal{L}(\Pi)e^{\Pi}$ is increasing in Π. On the other hand, $\mathcal{L}(\Pi) \leq \frac{e^{-\Pi}L(0)}{r}$ because

$$r\mathcal{L}(\Pi) = \min_q e^{-\Pi}L(q) + cq + \mathcal{L}'(\Pi)\pi(q) \leq e^{-\Pi}L(0).$$

Third, we show $q(\Pi) = 0$ for all $\Pi > \Pi_t$. It follows from $e^{-\Pi}L'(0) + c + \mathcal{L}'(\Pi)\pi'(0) > e^{-\Pi_t}L'(0) + c + \mathcal{L}'(\Pi_t)\pi'(0) \geq 0$ that 0 is the unique minimizer for $\min_q e^{-\Pi}L(q) + cq + \mathcal{L}'(\Pi)\pi(q)$. Hence, $q(\Pi) = 0$.

■

LEMMA 6 *If $\{q_t\}_{t\geq 0}$ is a production plan with $q_t \leq \bar{q}$, $\forall t \geq 0$, then*

$$Y \equiv \int_0^\infty e^{-rt-\Pi_t}q_t dt \leq \frac{\bar{q}}{r + \pi(\bar{q})},$$

where $\Pi_t \equiv \int_0^t \pi(q_s)ds$.

PROOF: Define

$$Y_t \equiv \int_t^\infty e^{-r(s-t)-(\Pi_s-\Pi_t)} q_s ds,$$

which satisfies the differential equation $\frac{dY_t}{dt} = (r+\pi(q_t))Y_t - q_t$. We have

$$\frac{dY_t}{dt} = (r+\pi(q_t))\left(Y_t - \frac{q_t}{r+\pi(q_t)}\right) \geq (r+\pi(q_t))\left(Y_t - \frac{\bar{q}}{r+\pi(\bar{q})}\right),$$

where the inequality follows from the fact that $\frac{q}{r+\pi(q)}$ is increasing in q. Gronwall's inequality then implies $Y_t - \frac{\bar{q}}{r+\pi(\bar{q})} \geq \left(Y_0 - \frac{\bar{q}}{r+\pi(\bar{q})}\right) e^{rt+\Pi_t}$. If, by contradiction, $Y = Y_0 > \frac{\bar{q}}{r+\pi(\bar{q})}$, then $\lim_{t\to\infty} Y_t - \frac{\bar{q}}{r+\pi(\bar{q})} \geq \lim_{t\to\infty}\left(Y_0 - \frac{\bar{q}}{r+\pi(\bar{q})}\right) e^{rt+\Pi_t} = \infty$. This contradicts the fact that $Y_t \equiv \int_t^\infty e^{-r(s-t)-(\Pi_s-\Pi_t)} q_s ds \leq \int_t^\infty e^{-r(s-t)} \bar{q} ds = \frac{\bar{q}}{r}$. ■

LEMMA 7 *In the direct mechanism, a domestic industry without a transition does not have the incentive to report one.*

PROOF: It follows from $q_t \leq p^{-1}(0)$ and Lemma 6 that

$$\int_t^\infty e^{-r(s-t)-(\Pi_s-\Pi_t)} q_s ds \leq \frac{p^{-1}(0)}{r+\pi(p^{-1}(0))}. \tag{32}$$

In the direct mechanism, suppose the domestic industry has no transition up to time t. The continuation utilities for a truth teller and a liar (who reports a transition at t) are, respectively,

$$\begin{aligned}
\int_t^\infty e^{-r(s-t)}(1-e^{-(\Pi_s-\Pi_t)}) c q_s ds &= \int_t^\infty e^{-r(s-t)} c q_s ds - c\int_t^\infty e^{-r(s-t)-(\Pi_s-\Pi_t)} q_s ds, \\
M_t - \frac{c p^{-1}(0)}{r+\pi(p^{-1}(0))} &= \int_t^\infty e^{-r(s-t)} c q_s ds - c\frac{p^{-1}(0)}{r+\pi(p^{-1}(0))}.
\end{aligned}$$

The first utility is higher than the second because of (32). ■

PROOF OF THEOREM 5: We do not need to study the domestic industry's strategy after the transition, because any feasible strategy is optimal. Next, we will show $\tilde{q}_t = q_t$ before the transition is optimal. In our implementation, the domestic industry does not receive any subsidy after time 0 and is fully responsible for its production cost.

Therefore the industry's objective is to minimize the present value of its production cost before transition.

$$\begin{array}{ll} \min\limits_{\tilde{q}_t} & \int_0^\infty e^{-rt-\tilde{\Pi}_t}\tilde{q}_t dt \\ s.t. & \tilde{q}_t \geq q_t, \quad \forall t \geq 0. \end{array}$$

Let $\{\hat{q}_t\}_{t\geq 0}$ denote the optimal solution in the above problem and define

$$\hat{Y}_t \equiv \int_t^\infty e^{-r(s-t)-(\hat{\Pi}_s-\hat{\Pi}_t)}\hat{q}_s ds,$$

which is weakly below Y_t since $\{\hat{q}_t\}_{t\geq 0}$ is optimal. First, we show that $\hat{Y}_t = Y_t, \forall t$. We have

$$\begin{aligned} \frac{d\hat{Y}_t}{dt} &= (r+\pi(\hat{q}_t))\left(\hat{Y}_t - \frac{\hat{q}_t}{r+\pi(\hat{q}_t)}\right) \\ &\leq (r+\pi(\hat{q}_t))\left(Y_t - \frac{q_t}{r+\pi(q_t)}\right) \\ &\leq (r+\pi(q_t))\left(Y_t - \frac{q_t}{r+\pi(q_t)}\right) = \frac{dY_t}{dt}, \end{aligned} \tag{33}$$

where the first inequality follows from $\hat{Y}_t \leq Y_t$ and $\frac{\hat{q}_t}{r+\pi(\hat{q}_t)} \geq \frac{q_t}{r+\pi(q_t)}$, and the second inequality follows from $Y_t \leq \frac{q_t}{r+\pi(q_t)}$, which follows from Lemma 6 and the fact that $q_s \leq q_t, \forall s \geq t$ in the direct mechanism. Since $\lim_{t\to\infty} q_t = 0$ implies $\lim_{t\to\infty} Y_t = 0$, it follows from (33) that

$$0 \leq \lim_{s\to\infty} \hat{Y}_s = \lim_{s\to\infty}\left(\hat{Y}_s - Y_s\right) \leq \hat{Y}_t - Y_t, \quad \forall t \geq 0,$$

which, together with $\hat{Y}_t \leq Y_t$, imply $\hat{Y}_t = Y_t, \forall t$.

Second, we show $\hat{q}_t = q_t, \forall t$. Function $(r+\pi(q))Y_t - q$ is decreasing in $q \geq q_t$ because its derivative with respect to q is negative:

$$\pi'(q)Y_t - 1 \leq \pi'(q)\frac{q_t}{r+\pi(q_t)} - 1 \leq \pi'(q_t)\frac{q_t}{r+\pi(q_t)} - 1 < 0.$$

It follows from the monotonicity of $(r+\pi(q))Y_t - q$ in q and $(r+\pi(\hat{q}_t))Y_t - \hat{q}_t = \frac{d\hat{Y}_t}{dt} =$

$\frac{dY_t}{dt} = (r+\pi(q_t))Y_t - q_t$ that $\hat{q}_t = q_t$. ■

PROOF OF THEOREM 6: First, the objective function $L^*(q) \equiv \frac{L(q)}{r+\pi(q)} + \frac{c}{r}q$ is convex in q because its derivative

$$(L^*)'(q) = \frac{L'(q)}{r+\pi(q)} - \frac{L(q)}{(r+\pi(q))^2}\pi'(q) + \frac{c}{r}$$

is monotonically increasing in q. Therefore, the first-order condition $(L^*)'(q^*) = 0$ is necessary and sufficient for finding q^*.

Second, by contradiction, suppose $q^* \geq q_0$. It follows from the monotonicity of q_t that $q^* > q_t$ for all $t > 0$. We have

$$\begin{aligned}
-L'(q^*) + \frac{L(q^*)}{r+\pi(q^*)}\pi'(q^*) &= \frac{r+\pi(q^*)}{r}c \\
&\geq c \\
&= -L'(q_0) + \left[\int_0^\infty e^{-rs-\Pi_s}L(q_s)ds\right]\pi'(q_0) \\
&> -L'(q_0) + \left[\int_0^\infty e^{-rs-\pi(q^*)s}L(q^*)ds\right]\pi'(q_0) \\
&= -L'(q_0) + \frac{L(q^*)}{r+\pi(q^*)}\pi'(q_0),
\end{aligned}$$

where the first equality follows from $(L^*)'(q^*) = 0$, the second equality follows from the first-order condition for q_0, and the second inequality follows from $q^* > q_t, \forall t > 0$. Since $-L'(q) + \frac{L(q^*)}{r+\pi(q^*)}\pi'(q)$ is decreasing in q, the above inequality implies $q^* < q_0$, contradicting the assumption of $q^* \geq q_0$. ■

PROOF OF THEOREM 7: In this proof, we first solve the HJB equation (15) for the linear example by "guess and verify." Conjecture the value function as

$$\mathcal{L}(\Pi) = \begin{cases} \frac{c}{r}\left(1 - \frac{\pi}{r+\pi}e^{\frac{r}{\pi}(\Pi-\bar{\Pi})}\right), & \text{if } \Pi < \bar{\Pi}; \\ \frac{\gamma}{r}e^{-\Pi}, & \text{if } \Pi \geq \bar{\Pi}, \end{cases}$$

where $\bar{\Pi} = \pi\bar{t}$. First, we verify (15) for $\Pi \geq \bar{\Pi}$. Since $\mathcal{L}'(\Pi) = -\frac{\gamma}{r}e^{-\Pi}$, we have

$$\min_q e^{-\Pi}\gamma(1-q) + cq + \mathcal{L}'(\Pi)\pi q \quad = \quad \min_q e^{-\Pi}\gamma(1-q) + cq - \frac{\gamma}{r}e^{-\Pi}\pi q$$

$$\begin{aligned} &= \min_q e^{-\Pi}\left(\gamma - \gamma q - \frac{\gamma}{r}\pi q\right) + rcq \\ &= e^{-\Pi}\gamma = r\mathcal{L}(\Pi). \end{aligned}$$

Second, we verify (15) for $\Pi < \bar{\Pi}$. Since $\mathcal{L}'(\Pi) = -\frac{c}{r+\pi}e^{\frac{r}{\pi}(\Pi-\bar{\Pi})}$, we have

$$\min_q e^{-\Pi}\gamma(1-q) + cq + \mathcal{L}'(\Pi)\pi q = \min_q e^{-\Pi}\gamma(1-q) + cq - \frac{c}{r+\pi}e^{\frac{r}{\pi}(\Pi-\bar{\Pi})}\pi q.$$

The derivative of the above with respect to q is $f(\Pi) \equiv c - e^{-\Pi}\gamma - \frac{c\pi}{r+\pi}e^{\frac{r}{\pi}(\Pi-\bar{\Pi})}$. We show that $f(\Pi) < 0$ for all $\Pi < \bar{\Pi}$. We have

$$\begin{aligned} f(\bar{\Pi}) &= 0, \\ f'(\bar{\Pi}) &= \left(e^{-\Pi}\gamma - \frac{cr}{r+\pi}e^{\frac{r}{\pi}(\Pi-\bar{\Pi})}\right)|_{\Pi=\bar{\Pi}} = 0. \end{aligned}$$

So it follows from the concavity of f that $f(\Pi) < 0$ for all $\Pi < \bar{\Pi}$. Hence the optimal $q = 1$ and

$$\min_q e^{-\Pi}\gamma(1-q) + cq - \frac{c}{r+\pi}e^{\frac{r}{\pi}(\Pi-\bar{\Pi})}\pi q = c - \frac{c\pi}{r+\pi}e^{\frac{r}{\pi}(\Pi-\bar{\Pi})} = r\mathcal{L}(\Pi).$$

In the above verification of the HJB equation, the optimal policy function is shown to be

$$q(\Pi) = \begin{cases} 1, & \text{if } \Pi < \bar{\Pi}; \\ 0, & \text{if } \Pi \geq \bar{\Pi}, \end{cases}$$

which implies (24). Plugging (24) into $M_t = \int_t^\infty e^{-r(s-t)}cq_s ds$ yields (26). ■

Appendix B

Suppose the government cares about both the consumers and the domestic industry, and puts a weight $\delta < 1$ on the latter. The optimal policy under public information ($q_t = q^{pub}, \forall t \geq 0$ in Theorem 1) remains unchanged: The optimal payoff of the domestic industry is still zero because the government puts a higher weight on the consumers.

Under private information, the government's objective function changes to

$$
\begin{aligned}
& -\int_0^{\infty} e^{-rt-\Pi_t}\left(L(q_t)+cq_t+\pi(q_t)(1-\delta)M_t\right)dt \\
= & -\int_0^{\infty} e^{-rt-\Pi_t}(L(q_t)+cq_t)dt-\int_0^{\infty}(1-e^{-\Pi_t})(1-\delta)e^{-rt}cq_t dt \\
= & -\int_0^{\infty} e^{-rt}(e^{-\Pi_t}(L(q_t)+\delta cq_t)+(1-\delta)cq_t)dt \\
= & -(1-\delta)\int_0^{\infty} e^{-rt}(e^{-\Pi_t}\tilde{L}(q_t)+cq_t),
\end{aligned}
$$

where $\tilde{L}(q) \equiv \frac{L(q)+\delta cq}{1-\delta}$. In other words, the government's problem with a positive weight on the domestic industry is equivalent to another problem in which the weight is zero but the social loss function increases from $L(q)$ to $\tilde{L}(q)$. Intuitively, the government provides more protection either with a positive weight on the domestic industry or with a higher social loss function. Our Theorems 2, 3, and 4 continue to hold.[14]

[14]Note that the parameter value of $\bar{c}$ in Theorems 3 and 4 has been changed by the new social loss function $\tilde{L}$.